INTRODUCTION TO ALGEBRAIC GROUPS: SYLLABUS

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This is the syllabus for the course "Introduction to Algebraic Groups", taught at the Weizmann Institute of Science in the first semester of the 2020-2021 academic year. Website: https://ivganev.github.io/teaching/algebraic-groups.html.

1. Overview

The theory of algebraic groups is central to various subjects in mathematics, ranging from number theory to mathematical physics. While the subject emerged from the analytic theory of Lie groups, the modern theory of algebraic groups is rooted in the principle that many groups of interest (including finite groups, classical groups, etc.) can be defined and understood using the powerful language of algebraic geometry. This principle provides a uniform approach to studying a wide variety of groups over arbitrary fields beyond the real and complex numbers encountered in the case of classical Lie groups.

In this introductory course, we exposit the rich theory of algebraic groups and supply useful tools to those interested in group theory, representation theory, number theory, algebraic geometry, and related subjects. The focus will be on linear (affine) algebraic groups over a field of characteristic zero, while touching on several other pertinent topics.

Specific material will include:

- Review of basic algebraic geometry and commutative algebra
- Structure of algebraic groups: Borel subgroups, parabolic subgroups, Jordan decomposition
- Homogeneous spaces and quotients
- Derivations, differentials, and Lie algebras
- Roots, weights, Weyl groups, and root datum
- Reductive groups and their classification in terms of root data
- Commutative algebraic groups and abelian varieties (time permitting)

2. Prerequisites

Basic algebra (groups, rings, and fields) is crucial. Some familiarity with or previous exposure to algebraic geometry and commutative algebra is necessary.

3. Textbook and resources

The course will follow:

- T.A. Springer, Linear algebraic groups (2nd edition), Modern Birkhäuser Classics, 1988 (link).

We may cover or reference material from the following textbooks:

- Armand Borel, Linear Algebraic Groups (link).
- James Humphreys, Linear Algebraic Groups (link).
- J. S. Milne course notes (link).

Useful references for algebraic geometry are:

- Ravi Vakil's course "Foundations of Algebraic Geometry."
- Robin Hartshorne, Algebraic Geometry.

4. Evaluation

The class will be pass/fail and the grade will be determined by assignments (50%) and the final exam (50%).

The assignment component consists of the submission of a total of ten exercises throughout the course. Eligible exercises are listed on the website. Discussion of the exercises with classmates is strongly encouraged, but I ask that every student write up their own assignment.

The final exam will consist of a two-week take-home portion and a short oral examination (15-20 minutes). The details and dates will be communicated via email to the mailing list for the course.